Modification of Attractiveness and Movement of the Firefly Algorithm for Resolution to Knapsack Problems

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***Abstract*—Knapsack problems pertain to selections of a number of items stored in order to obtain optimal storage. A container can accommodate these items with weight and values through the consideration of the capacity of the storage media. In this research, strategy on attractiveness and movement of the firefly algorithm was proposed to solve knapsack problems. This algorithm was tested through the comparison of the others, namely the original firefly algorithm, the firefly algorithm with attractiveness modification, the firefly algorithm with movement modification, and the firefly algorithm with a modified combination of attractiveness and movement. Applying each of them, there are differences of time and results of completion. Compared to the original firefly algorithm, the one with a modified combination of attractiveness and movement has the best convergence behavior and global optimization efficiency. It is found that the more iterations and the number of fireflies are, the longer the processing time will be. Despite this, the original firefly algorithm requires short time.**

Keywords—Firefly Algorithm, Movement, Attractiveness, Optimization, Knapsack Problems

I. Introduction

Knapsack is a bag where certain items are inserted. It can only store some of them with total weight which is less than or equal to its capacity. Knapsack problems are optimization of selecting items which can be inserted into the container with limited space or capacity [1][2]. With this optimization, it is expected that maximum profits can be earned.

Inserted items have weight and values used to determine selection priorities [1][2]. The container also has constant values becoming the barriers. Hence, a method should be implemented to break them to produce optimal outcomes without exceeding the capacity [1][2].

Currently, there are numerous algorithms which can be used to solve knapsack optimization problems such as the brute force algorithm [3][4] with an optimal property and long process time, the greedy algorithm [5] with high speed and a nonoptimal property, and the dynamic programming algorithm [4] with an optimal property and rather fast process time.

Another optimization algorithm applies swarm intelligence [6][7]. It refers to an artificial intelligence technique which is based on collective behavior in a decentralized and self-organizing system [6]. The population of members in the form of simple agents interacting locally with fellow members and the environment are covered [6][7].

The firefly algorithm is one of optimization algorithms applied to solve knapsack problems [8]. Previous studies were on the binary firefly algorithm [8][9], optimization with modification [10], and resolution to resource allocation [11]. In Tilahun's work, the search mechanism was modified to improve FA performance [18]. Other research papers present a new Firefly algorithm that uses a hybrid strategy to achieve excellent optimization performance. That is, an adaptive parameter strategy that dynamically changes step coefficients, a modified search strategy, eliminates the notion of attraction and uses a stochastic attraction model. replace the model with Full of original attractiveness [19]. Various knapsack problems were solved [8][9][11][12]. A global firefly algorithm is proposed to solve the randomly changing time-varying knapsack problem [20]. The purpose of this study was to propose a modified combination of attractiveness and movement of the firefly algorithm for resolution to them. Continuously optimizing the complex problems, previous studies conducted modification [10][13][14]. Disadvantages of using the firefly algorithm are poor convergence behavior and a greater tendency of falling to local optimality. Modification made increases attractiveness [13][14].

II. Methods

A. Research Method

This was an experimental study focusing on three stages such as library research and data collection, comparison of the firefly algorithm and variants, and analysis of test results. Scientific literature in relation to such the algorithm and the knapsack problems was referred to. Dataset 01 of knapsack problems could be downloaded at https://people.sc.fsu.edu/~jburkardt/datasets/knapsack_01/knapsack_01.html. In terms of comparison, the variant of the firefly algorithm producing the most optimal results was the best one. Specifically, performance of the number of iterations and fireflies, and optimal solutions to the dataset of knapsack problems with values 0-1 were compared. To perform more optimal comparison, it was requisite to carry out examination of the best dataset results by repeating the tests several times. Furthermore, the number of optimal solutions, mean, and standard deviation, and computation time should be considered to determine the best firefly algorithm. Here finding the optimal values or profits of combined items with certain limitations was the goal.

B. Knapsack Problems

Knapsack is a bag where certain items are inserted [2][3]. It can only store some of them with total weight which is less than or equal to its capacity.

There are several types of knapsack problems such as a) 0/1 knapsack problem with only an available unit of each item, b) fractional knapsack problem with items that can only be partially carried, c) bounded knapsack problem with limited N units of items, and d) unbounded knapsack problem with at least two or even unlimited items [1][2]. The former is most frequently and commonly used. It restricts possible choices made between 1 and 0 [1].

The total size or capacity of knapsack problems is symbolized by V. Different types (r) are included. Certain items have w_r weight and p_r values. In this case, Equation 1 shows objective functions to maximize knapsack profits. Equation 2, however, represents the restrictions of capacity. Finally, Equation 3 imposes the ones of variable decisions [1].

$$maximize\ \sum_{r \in R} p_r x_r \quad (1)$$

$$\sum_{r \in R} w_r x_r \leq V \quad (2)$$

$$x_r \in \{0,1\}, r \in R \quad (3)$$

where $R = \{1, 2, ..., r\}$ is the set of items, p_r represents profits of item r, w_r is the resource consumption of item r in the container, V is the capacity of the knapsack, x_r is a binary variable representing whether item r is assigned to the knapsack (x_r =1 if this item is assigned, x_r =0 if it is not).

C. Firefly Algorithm

The firefly algorithm refers to a metaheuristic algorithm inspired by the flickering behavior of fireflies [8][9][11][15]. Basic functions of flashing light are to attract the attention of the others, have communication, and attract the prey [15][16]. Principally, in this algorithm, a) regardless of the similar gender, all fireflies are attracted to each other; b) the attractiveness is proportional to the brightness of the blinking light. Therefore, fireflies with lower brightness levels will be attracted and move to the others with higher ones. These levels decrease, the distance increases, and there is light absorption due to the air. Provided that there is none of the brightest light in the population, the whole fireflies will move at random; and c) the brightness or light intensity is influenced or determined by values of objective functions of given problems [16]. For maximization, it is proportional to such values.

Generally, formulation of this algorithm is presented with analytical and mathematical modeling to solve problems with function equivalence objectives. Outcomes are compared with alternative techniques proposed in the literature to indicate that optimal and correct solutions can be generated. The firefly algorithm is easy to implement, continues to grow, and can assist resolution to various optimization problems [16]. The following is its pseudocode [15][16]:

```
Initialize the firefly algorithm parameters, such as the number of
fireflies (n), β0, γ, α, and the maximal number of generations
(iterations, MaxGen).
Set objective functions f(x), x = (x1,…,xd)T.
Generate an initial population of fireflies xi (i = 1,2,…,n).
While k < MaxGen //(k = 1:MaxGen)
For i = 1 : n //all fireflies n
    For j = 1 : n
        If (Ij > Ii)
            Move firefly i to the another one j in Dimension d
        Else
            Move firefly i at random
        End If
        Get attractiveness values, varying with distance r
        Find new solutions and update light intensity values
    End for j
End for i
Rank fireflies and find current best.
End While
Finding fireflies with the highest light intensity.
```

The principle of light intensity is an indication determining whether fireflies move or not. Their intensity is described in Equation 4 [15][16]:

$$I = \frac{1}{f(x)} \quad (4)$$

Where *f(x)* is the total distance. Distribution of fireflies is measured through Euclidean distance calculation represented in Equation 5 [15][16]:

$$r_{ij} = \sqrt{\sum_{R=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (5)$$

Where:
r_{ij} = distance between fireflies *i* and *j*
$x_{i,k}$ = dimension value *k* of firefly *i*
$x_{j,k}$= dimension value *k* of firefly *j*

The principle of movement is influenced by firefly attractiveness to the brighter light (see Figure 1). This movement is represented in Equation 6 [15][16]:

$$x_i = x_i + \beta_0 e^{-\gamma r^2 ij} (x_i - x_t) + \alpha\ sign \left[rand - \frac{1}{2}\right] \quad (6)$$

Where:
x_i= position of firefly *i*
β_0= attractiveness of fireflies at position 0
γ = coefficient of light absorption
α = coefficient of random numbers
t = number of iterations
r_{ij} = distance between fireflies *i* and *j*

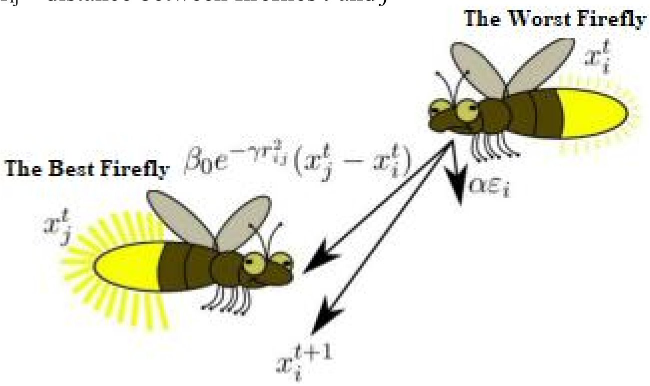

Fig. 1. Mechanism of the Firefly Algorithm [4]

D. Proposed Method

In this study, modification was made to the attractiveness and movement of fireflies. The rationale is that fireflies will move to follow other fireflies with brighter intensity. Thus, there is a possibility of moving to them. Previous studies used the Levy flight distribution by providing random signs as direction markers [15]. This study, nevertheless, changed the concept of changing movement values by replacing the Levy flight with small and random ones (see Equation 7)

$$x_i = x_i + \beta_0 e^{-\gamma r^2 ij} (x_i - x_t) + \alpha\ sign \left[rand - \frac{1}{2}\right] + L \quad (6)$$

Where:

$$L = rand * 0.1 + 0.1 \quad (7)$$

Attractiveness triggering the movement of fireflies from location x_i to location x_t at the time which is less than r is based on the light intensity of adjacent fireflies (see Equation 8) [16]. In this study, attractiveness changes of various fireflies are provided (see Equations 9 and 10).

$$\beta = \beta_0 e^{-\gamma r^2} \quad (8)$$

Where β_0 is attractiveness when there is no distance among fireflies *(r = 0)* and $\gamma \in [0, \infty]$ is the light absorption coefficient.

$$\beta = \beta_0 e^{-\gamma r^2} . \Delta\beta \quad (9)$$

$$\Delta\beta = \frac{1}{2} * \frac{1 - I(x_i)}{(x_i - x_t)} \quad (10)$$

Where:

$I(x_i)$ = intensity of fireflies
x_i = position of firefly *i*
t = number of iterations

III. Experimental Results

This research solved the 0/1 knapsack problem through a number of datasets retrieved from https://people.sc.fsu.edu/~jburkardt/datasets/knapsack_01/knapsack_01.html. Each dataset was assigned a knapsack, a fixed capacity of C, and a list of N items having weight W and profits P (see Table 1).

TABLE I. Dataset of 0/1 Knapsack Problem

Dataset	Number of Items	Capacity	Optimal Value
P01	10	165	309
P02	5	26	51
P03	6	190	150
P04	7	50	107
P05	8	104	900
P06	7	190	1735
P07	15	750	1458
P08	24	6404180	13549094

Source: https://people.sc.fsu.edu/~jburkardt/datasets/knapsack_01/knapsack_01.html

Trials were implemented by using a computer with specifications of processor Intel(R) Core(TM) i5-7200U, CPU @ 2.50GHz 2.70GHz, and 12 GB RAM. Anaconda with the python, a programming language, became a tool for software development.

Performance of compared firefly algorithms, namely the original firefly algorithm, the firefly algorithm with attractiveness modification, the firefly algorithm with movement modification, and the firefly algorithm with a modified combination of attractiveness and movement for resolution to knapsack problems was provided. Testing of the number of fireflies and iterations was conducted by finding nearly optimal solutions from several knapsack datasets. Examination of performance was, however, carried out by comparing the values of the number of appropriate and optimal solutions generated by the original firefly algorithm and the proposed modification of the firefly algorithm.

The first trial scenario was actualized to determine performance of all firefly algorithms based on the outcomes of solving the knapsack problems from a number of datasets. Experiments of each dataset with the same parameters were repeated 10 times. Optimal solutions of trials, patterns formed, and the processing time became references. The trial data used were 8 datasets and 0/1 knapsack problem comprising P01, P02, P03, P04, P05, P06, P07, and P08. Parameters, on the other hand, consisting of the number of fireflies (N = 50), the number of iterations (K = 50), a random coefficient (α = 1), the attractiveness value when there is no distance among fireflies (β0 = 1), and the coefficient of light absorption (γ = 0.5) were provided in Table 1.

TABLE II. Examination Result of the Dataset

Dataset	Measurement	Original FA	FA of Attractiveness and Movement	Attractiveness FA	Movement FA
P01	Mean	293	305.8	302.6	302.6
	StdDev	16	9.6	12.8	12.8
	Max	309	309	309	309
	Min	277	277	277	277
P02	Mean	51	51	51	51
	StdDev	0	0	0	0
	Max	51	51	51	51
	Min	51	51	51	51
P03	Mean	151.5	150	150	150
	StdDev	2.291288	0	0	0
	Max	155	150	150	150
	Min	150	150	150	150
P04	Mean	107	107	107	107
	StdDev	0	0	0	0
	Max	107	107	107	107
	Min	107	107	107	107
P05	Mean	895.6	899.3	894.7	899.2
	StdDev	6.359245	1.552417	7.681797	0.979796
	Max	900	900	900	900
	Min	883	895	883	898
P06	Mean	1712.3	1726	1717	1712
	StdDev	17.00029	15.4013	18.0333	18.13284
	Max	1735	1745	1735	1735
	Min	1682	1692	1688	1682
P07	Mean	1431.4	1445.7	1437.7	1440.3
	StdDev	7.565712	3.9	7.34915	5.197115
	Max	1444	1452	1454	1449
	Min	1415	1437	1430	1434
P08	Mean	13211679	13199650	13198131	13220431
	StdDev	84843.2	59693.15	116193.7	85336.22
	Max	13399518	13292007	13458609	13334101
	Min	13114239	13122592	13040689	13092294

Pertaining to the comparison of the standard deviation and the maximum values, nonetheless, the firefly algorithm with a modified combination of attractiveness and movement was almost an optimal solution despite the fact that it was weaker than the original firefly algorithm. The test results of datasets indicated that all algorithms had different values and some tended to provide local, optimum solutions. This reinforced the fact that the firefly algorithm had disadvantages of convergence behavior and easily fell to local optimality [13]. Modification of movement was, therefore, required to increase attractiveness through reconstruction and enhancement of items. Overall, the firefly algorithm with a modified combination of attractiveness and movement had the best convergence behavior and global optimization efficiency.

TABLE III. Examination Result of an Optimal Solution

Data set	Number of Optimal Solutions			
	Original FA	**FA of Attractiveness and Movement**	**Attractiveness FA**	**Movement FA**
P01	5	9	8	8
P02	10	10	10	10
P03	7	10	10	10
P04	10	10	10	10
P05	3	8	6	6
P06	1	5	4	3

P07	0	0	0	0
P08	0	0	0	0

Table 3 reflected that in terms of P02 and P04, the firefly algorithm and its modification could similarly generate optimal solutions. Conversely, this did not apply to P07 and P08 (see Figure 2) as all algorithms were trapped in the local optimality and there were computational influences of datasets. Apparently, variation of more than fifteen items and large profit values was found.

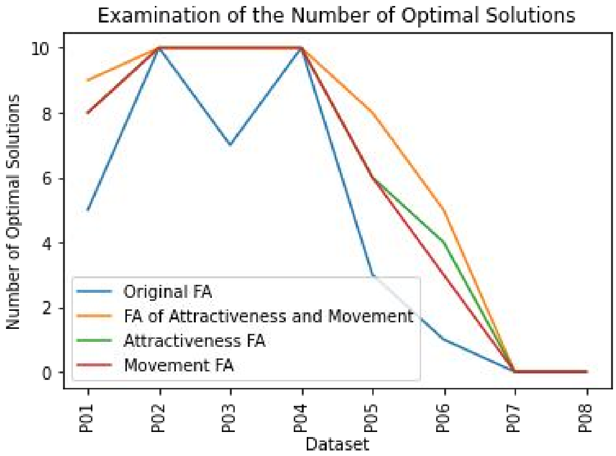

Fig. 2. Graphic of the Number of Optimal Solutions

TABLE IV. EXAMINATION RESULT OF DATASET TIME

Dataset	Measurement	Original FA (in Seconds)	FA of Attractiveness and Movement (in Seconds)	Attractiveness FA (in Seconds)	Movement FA (in Seconds)
P01	Mean	8.6407	16.7309	15.8498	4.4624
	StdDev	0.8712	2.3388	3.0681	1.1313
	Max	9.8646	20.8524	23.3315	6.6293
	Min	7.2437	13.3552	12.2682	2.1831
P02	Mean	0.9587	9.2727	10.7194	1.8589
	StdDev	0.4374	0.7046	1.4407	0.3566
	Max	1.9298	10.1249	13.3822	2.5023
	Min	0.3920	8.3128	8.6658	1.3744
P03	Mean	3.1312	10.0826	11.2541	2.9915
	StdDev	0.2658	0.5392	1.3317	0.2602
	Max	3.6762	11.1611	12.8177	3.354
	Min	2.7068	9.344	8.4553	2.5103
P04	Mean	5.3655	10.1904	12.773	4.5139
	StdDev	0.1558	1.3152	1.923	0.6267
	Max	5.7167	11.3876	14.594	5.3088
	Min	5.1333	6.5325	8.6828	3.1567
P05	Mean	6.6727	10.1649	11.4134	4.8036
	StdDev	0.5869	2.5878	3.0182	0.8241
	Max	8.0266	13.1518	19.5218	6.1665
	Min	5.9701	4.2307	8.6309	3.0678
P06	Mean	4.638	12.1016	10.4081	3.7906
	StdDev	0.6056	0.8756	1.0409	0.9998
	Max	5.2609	13.8799	12.691	5.7446
	Min	3.1376	10.8608	8.978	2.1163
P07	Mean	14.9052	22.1725	23.2448	14.0309
	StdDev	0.8839	1.2803	2.8863	0.4181
	Max	17.4807	24.1396	27.3173	14.6216
	Min	14.063	20.5382	18.5884	13.1129
P08	Mean	22.8531	32.1047	37.8256	23.2256
	StdDev	0.7639	1.5238	3.9081	0.5952
	Max	24.2439	34.4468	45.6211	24.3459
	Min	21.3064	28.9971	30.4127	22.108

Examining the datasets of the processing time, it was found that the original firefly algorithm and the firefly algorithm with movement modification were relatively the same (see Table 4). The reason was that the simple computation in the modification section was applied. Meanwhile, the firefly algorithm with attractiveness modification and the one with a modified combination of both had relatively longer processing time in finding the optimal solutions (see Figure 3). The complexity of equations and the number of item variations in the datasets could affect the calculation.

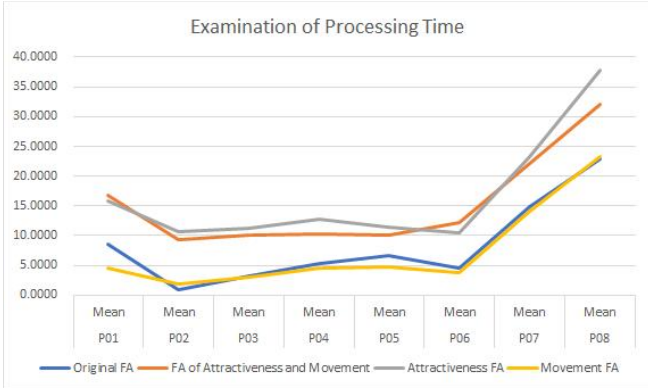

Fig. 3. Graphic of Average Processing Time of Dataset Examination

Following this, the second examination of the number of fireflies was conducted to obtain the optimal one used in the scenario. P02 was used as knapsack data. This dataset was selected as outcomes of the first trial revealed that an optimal solution generated by each algorithm was similar. The numbers of fireflies used were respectively 50, 100, 150, 200, 250, 300, 350, 400, 450 and 500. The parameters covered the number of iterations K = 50, the coefficient of the random parameter α = 1, the value of attractiveness when there was no distance among fireflies β_0 = 1, and the coefficient of light absorption γ = 0.5. Test results of the number of fireflies in modified algorithms were presented in Figure 4.

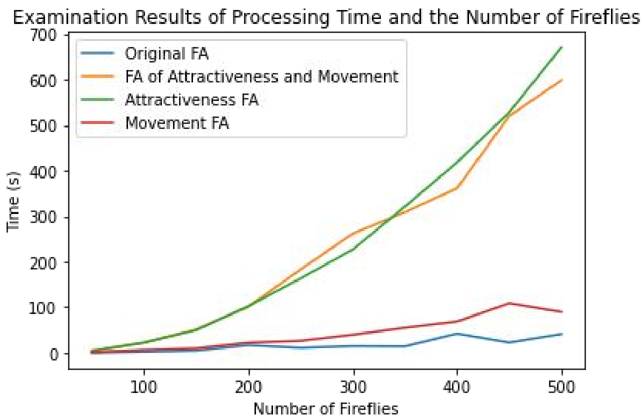

Fig. 4. Examination of the Number of Fireflies

Optimal solutions generated by the four algorithms were the same for P02. Referring to the graphic of the examination results of the number of iterations in Figure 1, the firefly algorithm with a modified combination of attractiveness and movement emphasized that more fireflies determined longer processing time, while the original one required short time.

The third examination of the numbers of iterations (respectively 50, 100, 150, 200, 250, 300, 350, 400, 450, and 500) was conducted to obtain the optimal one used in the scenario. P02 was used as the knapsack data. The parameters consisted of the number of fireflies N = 50, the coefficient of the random parameter α = 0.5, the value of attractiveness when there was no distance among fireflies $\beta0$ = 1, and the coefficient of light absorption γ = 0.5. Test results of the number of iterations in modified firefly algorithms were shown in Figure 5.

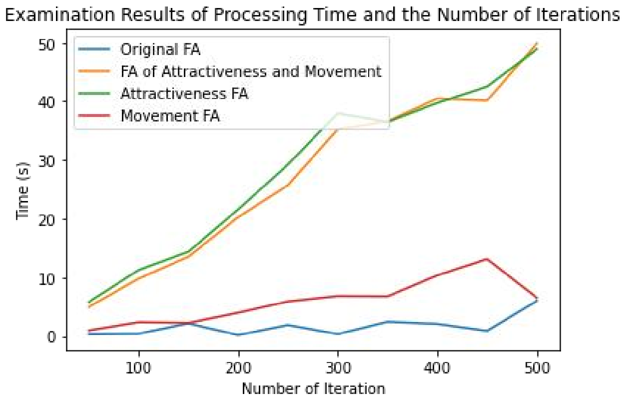

Fig. 5. Examination of the Number of Iterations

Optimal solutions were similarly obtained through the four algorithms with P02. Figure 2 indicated trial results of the number of iterations through a modified combination of attractiveness and movement. It was noted that the more iterations there were, the longer the processing time would be. Despite this, the original firefly algorithm required short time.

CONCLUSION

To conclude, by using a complex dataset, almost all of the firefly algorithms and their variants are trapped in the local optimality due to large variety of items in it. The processing time of the original firefly algorithm and the firefly algorithm with movement modification relatively have the same processing time. Meanwhile, the one of the firefly algorithm with attractiveness modification and a combination of attractiveness and movement is relatively longer in finding the optimal solutions. Compared to the others, the firefly algorithm with this combination has the best convergence behavior and global optimization efficiency. Obviously, the more iterations and the number of fireflies are, the longer the processing time will be. Despite this, the original firefly algorithm requires short time. For further research, there should be hybridization of firefly algorithms and others. Accordingly, enhanced searches for optimal solutions and reduced processing time are actualized. Additionally, other datasets and types of knapsack problems can be used.

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